Preface

Welcome to *Title*, an OpenStax resource. This textbook was written to increase student access to high-quality learning materials, maintaining highest standards of academic rigor at little to no cost.

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About OpenStax Resources

Customization

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Format

You can access this textbook for free in web view or PDF through openstax.org, and for a low cost in print.

About *Title*

Title is designed to meet the scope and sequence requirements of [include typical course title, and semester length of course if pertinent]. [List notable elements of the book that are important for this subject; specify the targeted student audience if not for a general audience. Blurb to be approved by OSX marketing team.]

Coverage and Scope

Title follows a nontraditional approach in its presentation of content. Building on the content in *Prealgebra*, the material is presented as a sequence of small steps so that students gain confidence in their ability to succeed in the course. The order of topics was carefully planned to emphasize the logical progression through the course and to facilitate a thorough understanding of each concept. As new ideas are presented, they are explicitly related to previous topics.

- **Chapter 1: Foundations**
  Chapter 1 reviews arithmetic operations with whole numbers, integers, fractions, and decimals, to give the student a solid base that will support their study of algebra.
- **Chapter 2: Solving Linear Equations and Inequalities**
  In Chapter 2, students learn to verify a solution of an equation, solve equations using the Subtraction and Addition Properties of Equality, solve equations using the Multiplication and Division Properties of Equality, solve equations with variables and constants on both sides, use a general strategy to solve linear equations, solve equations with fractions or decimals, solve a formula for a specific variable, and solve linear inequalities.
- **Chapter 3: Math Models**
  Once students have learned the skills needed to solve equations, they apply these skills in Chapter 3 to solve word and number problems.
- **Chapter 4: Graphs**
  Chapter 4 covers the rectangular coordinate system, which is the basis for most consumer graphs. Students learn to plot points on a rectangular coordinate system, graph linear equations in two variables, graph with intercepts, understand slope of a line, use the slope-intercept form of an equation of a line, find the equation of a line, and create graphs of linear inequalities.

Key Features and Boxes

Examples Each learning objective is supported by one or more worked examples, which demonstrate the problem-solving approaches that students must master. Typically, we include multiple examples for each learning

objective to model different approaches to the same type of problem, or to introduce similar problems of increasing complexity.

All examples follow a simple two- or three-part format. First, we pose a problem or question. Next, we demonstrate the solution, spelling out the steps along the way. Finally (for select examples), we show students how to check the solution. Most examples are written in a two-column format, with explanation on the left and math on the right to mimic the way that instructors “talk through” examples as they write on the board in class.

Test H4

Test

Additional Resources

Student and Instructor Resources

We’ve compiled additional resources for both students and instructors, including Getting Started Guides, [other resources dependent on book]. Instructor resources require a verified instructor account, which can be requested on your openstax.org log-in. Take advantage of these resources to supplement your OpenStax book.

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Demand, Supply, and Efficiency
By the end of this section, you will be able to:

- Contrast consumer surplus, producer surplus, and social surplus
- Explain why price floors and price ceilings can be inefficient
- Analyze demand and supply as a social adjustment mechanism

The familiar demand and supply diagram holds within it the concept of economic efficiency. One typical way that economists define efficiency is when it is impossible to improve the situation of one party without imposing a cost on another. Conversely, if a situation is inefficient, it becomes possible to benefit at least one party without imposing costs on others.

Efficiency in the demand and supply model has the same basic meaning: The economy is getting as much benefit as possible from its scarce resources and all the possible gains from trade have been achieved. In other words, the optimal amount of each good and service is being produced and consumed.

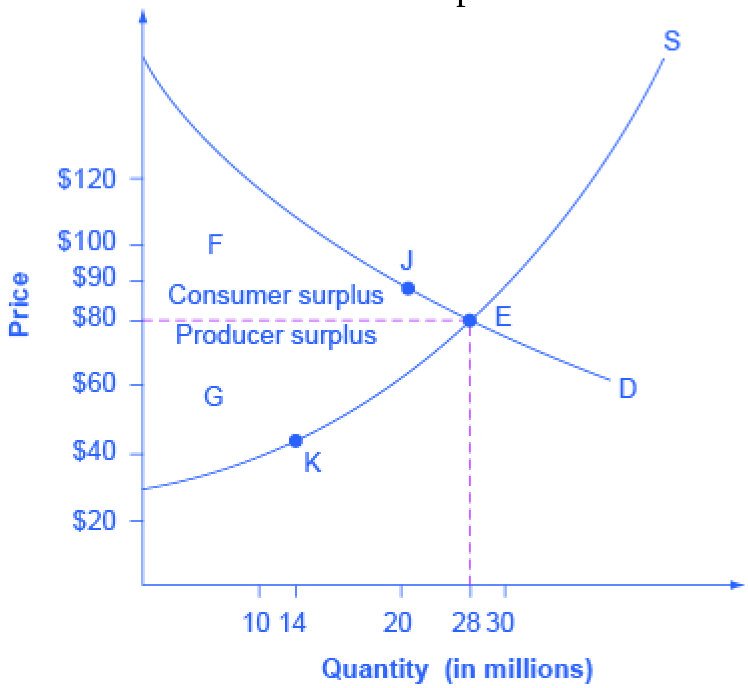
Consumer Surplus, Producer Surplus, Social Surplus

Consider a market for tablet computers, as shown in [link]. The equilibrium price is $80 and the equilibrium quantity is 28 million. To see the benefits to consumers, look at the segment of the demand curve above the equilibrium point and to the left. This portion of the demand curve shows that at least some demanders would have been willing to pay more than $80 for a tablet.

For example, point J shows that if the price was $90, 20 million tablets would be sold. Those consumers who would have been willing to pay $90 for a tablet based on the utility they expect to receive from it, but who were able to pay the equilibrium price of $80, clearly received a benefit beyond what they had to pay for. Remember, the demand curve traces consumers' willingness to pay for different quantities. The amount that individuals would have been willing to pay, minus the amount that they actually paid, is

called **consumer surplus**. Consumer surplus is the area labeled F—that is, the area above the market price and below the demand curve.

Consumer and Producer Surplus



The somewhat triangular area labeled by F shows the area of consumer surplus, which shows that the equilibrium price in the market was less than what many of the consumers were willing to pay. Point J on the demand curve shows that, even at the price of $90, consumers would have been willing to purchase a quantity of 20 million. The somewhat triangular area labeled by G shows the area of producer surplus, which shows that the equilibrium price received in the market was more than what many of the producers were willing to accept for their products. For example, point K on the supply curve shows that at a price of $45, firms would have been willing to supply a quantity of 14 million.

The supply curve shows the quantity that firms are willing to supply at each price. For example, point K in [link] illustrates that, at $45, firms would still have been willing to supply a quantity of 14 million. Those producers who would have been willing to supply the tablets at $45, but who were instead able to charge the equilibrium price of $80, clearly received an extra benefit beyond what they required to supply the product. The amount that a seller is paid for a good minus the seller's actual cost is called **producer surplus**. In [link], producer surplus is the area labeled G—that is, the area between the market price and the segment of the supply curve below the equilibrium.

The sum of consumer surplus and producer surplus is **social surplus**, also referred to as **economic surplus** or **total surplus**. In [link], social surplus would be shown as the area F + G. Social surplus is larger at equilibrium quantity and price than it would be at any other quantity. This demonstrates the economic efficiency of the market equilibrium. In addition, at the efficient level of output, it is impossible to produce greater consumer surplus without reducing producer surplus, and it is impossible to produce greater producer surplus without reducing consumer surplus.

Inefficiency of Price Floors and Price Ceilings

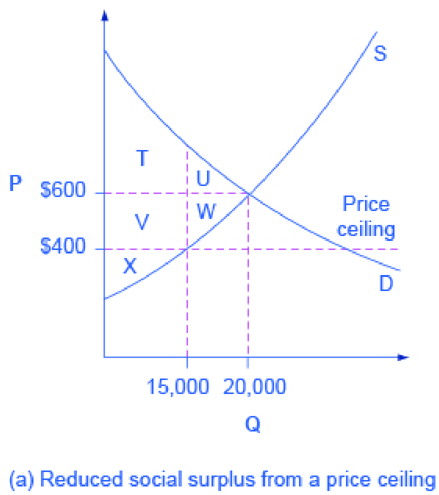
The imposition of a price floor or a price ceiling will prevent a market from adjusting to its equilibrium price and quantity, and thus will create an inefficient outcome. But there is an additional twist here. Along with creating inefficiency, price floors and ceilings will also transfer some consumer surplus to producers, or some producer surplus to consumers.

Imagine that several firms develop a promising but expensive new drug for treating back pain. If this therapy is left to the market, the equilibrium price will be $600 per month and 20,000 people will use the drug, as shown in [link] (a). The original level of consumer surplus is T + U and producer surplus is V + W + X. However, the government decides to impose a price ceiling of $400 to make the drug more affordable. At this price ceiling, firms in the market now produce only 15,000.

As a result, two changes occur. First, an inefficient outcome occurs and the total surplus of society is reduced. The loss in social surplus that occurs when the economy produces at an inefficient quantity is called **deadweight loss**. In a very real sense, it is like money thrown away that benefits no one. In [link] (a), the deadweight loss is the area U + W. When deadweight loss exists, it is possible for both consumer and producer surplus to be higher, in this case because the price control is blocking some suppliers and demanders from transactions they would both be willing to make.

A second change from the price ceiling is that some of the producer surplus is transferred to consumers. After the price ceiling is imposed, the new consumer surplus is T + V, while the new producer surplus is X. In other words, the price ceiling transfers the area of surplus (V) from producers to consumers. Note that the gain to consumers is less than the loss to producers, which is just another way of seeing the deadweight loss.

Efficiency and Price Floors and Ceilings



(a) The original equilibrium price is $600 with a quantity of 20,000. Consumer surplus is T + U, and producer surplus is V + W + X. A price ceiling is imposed at $400, so firms in the market now produce only a quantity of 15,000. As a result, the new consumer surplus is T + V, while the new producer surplus is X. (b) The original equilibrium is $8 at a quantity of 1,800. Consumer surplus is G + H + J, and producer surplus is I + K. A price floor is imposed at $12, which means that quantity demanded falls to 1,400. As a result, the new consumer surplus is G, and the new producer surplus is H + I.

[link] (b) shows a price floor example using a string of struggling movie theaters, all in the same city. The current equilibrium is $8 per movie ticket, with 1,800 people attending movies. The original consumer surplus is G + H + J, and producer surplus is I + K. The city government is worried that movie theaters will go out of business, reducing the entertainment options available to citizens, so it decides to impose a price floor of $12 per ticket. As a result, the quantity demanded of movie tickets falls to 1,400. The new consumer surplus is G, and the new producer surplus is H + I. In effect, the price floor causes the area H to be transferred from consumer to producer surplus, but also causes a deadweight loss of J + K.

This analysis shows that a price ceiling, like a law establishing rent controls, will transfer some producer surplus to consumers—which helps to explain why consumers often favor them. Conversely, a price floor like a guarantee that farmers will receive a certain price for their crops will transfer some consumer surplus to producers, which explains why producers often favor them. However, both price floors and price ceilings block some transactions that buyers and sellers would have been willing to make, and creates deadweight loss. Removing such barriers, so that prices and quantities can adjust to their equilibrium level, will increase the economy's social surplus.

Demand and Supply as a Social Adjustment Mechanism

The demand and supply model emphasizes that prices are not set only by demand or only by supply, but by the interaction between the two. In 1890, the famous economist Alfred Marshall wrote that asking whether supply or demand determined a price was like arguing "whether it is the upper or the under blade of a pair of scissors that cuts a piece of paper." The answer is that both blades of the demand and supply scissors are always involved.

The adjustments of equilibrium price and quantity in a market-oriented economy often occur without much government direction or oversight. If the coffee crop in Brazil suffers a terrible frost, then the supply curve of

coffee shifts to the left and the price of coffee rises. Some people—call them the coffee addicts—continue to drink coffee and pay the higher price. Others switch to tea or soft drinks. No government commission is needed to figure out how to adjust coffee prices, which companies will be allowed to process the remaining supply, which supermarkets in which cities will get how much coffee to sell, or which consumers will ultimately be allowed to drink the brew. Such adjustments in response to price changes happen all the time in a market economy, often so smoothly and rapidly that we barely notice them.

Think for a moment of all the seasonal foods that are available and inexpensive at certain times of the year, like fresh corn in midsummer, but more expensive at other times of the year. People alter their diets and restaurants alter their menus in response to these fluctuations in prices without fuss or fanfare. For both the U.S. economy and the world economy as a whole, markets—that is, demand and supply—are the primary social mechanism for answering the basic questions about what is produced, how it is produced, and for whom it is produced.

Note:
Why Can We Not Get Enough of Organic?
Organic food is grown without synthetic pesticides, chemical fertilizers or genetically modified seeds. In recent decades, the demand for organic products has increased dramatically. The Organic Trade Association reported sales increased from $1 billion in 1990 to $35.1 billion in 2013, more than 90% of which were sales of food products.

Why, then, are organic foods more expensive than their conventional counterparts? The answer is a clear application of the theories of supply and demand. As people have learned more about the harmful effects of chemical fertilizers, growth hormones, pesticides and the like from large-scale factory farming, our tastes and preferences for safer, organic foods have increased. This change in tastes has been reinforced by increases in income, which allow people to purchase pricier products, and has made organic foods more mainstream. This has led to an increased demand for organic foods. Graphically, the demand curve has shifted right, and we

have moved up the supply curve as producers have responded to the higher prices by supplying a greater quantity.

In addition to the movement along the supply curve, we have also had an increase in the number of farmers converting to organic farming over time. This is represented by a shift to the right of the supply curve. Since both demand and supply have shifted to the right, the resulting equilibrium quantity of organic foods is definitely higher, but the price will only fall when the increase in supply is larger than the increase in demand. We may need more time before we see lower prices in organic foods. Since the production costs of these foods may remain higher than conventional farming, because organic fertilizers and pest management techniques are more expensive, they may never fully catch up with the lower prices of non-organic foods.

As a final, specific example: The Environmental Working Group’s “Dirty Dozen” list of fruits and vegetables, which test high for pesticide residue even after washing, was released in April 2013. The inclusion of strawberries on the list has led to an increase in demand for organic strawberries, resulting in both a higher equilibrium price and quantity of sales.

Consumer surplus is the gap between the price that consumers are willing to pay, based on their preferences, and the market equilibrium price. Producer surplus is the gap between the price for which producers are willing to sell a product, based on their costs, and the market equilibrium price. Social surplus is the sum of consumer surplus and producer surplus. Total surplus is larger at the equilibrium quantity and price than it will be at any other quantity and price. Deadweight loss is loss in total surplus that occurs when the economy produces at an inefficient quantity.

Exercise:

Problem:

Does a price ceiling increase or decrease the number of transactions in a market? Why? What about a price floor?

Solution:

Assuming that people obey the price ceiling, the market price will be below equilibrium, which means that Qd will be more than Qs. Buyers can only buy what is offered for sale, so the number of transactions will fall to Qs. This is easy to see graphically. By analogous reasoning, with a price floor the market price will be above the equilibrium price, so Qd will be less than Qs. Since the limit on transactions here is demand, the number of transactions will fall to Qd. Note that because both price floors and price ceilings reduce the number of transactions, social surplus is less.

Exercise:

Problem:

If a price floor benefits producers, why does a price floor reduce social surplus?

Solution:

Because the losses to consumers are greater than the benefits to producers, so the net effect is negative. Since the lost consumer surplus is greater than the additional producer surplus, social surplus falls.

Exercise:

Problem:

What is consumer surplus? How is it illustrated on a demand and supply diagram?

Exercise:

Problem:

What is producer surplus? How is it illustrated on a demand and supply diagram?

Exercise:

Problem:

What is total surplus? How is it illustrated on a demand and supply diagram?

Exercise:

Problem:

What is the relationship between total surplus and economic efficiency?

Exercise:

Problem: What is deadweight loss?

Exercise:

Problem:

What term would an economist use to describe what happens when a shopper gets a “good deal” on a product?

Exercise:

Problem: Explain why voluntary transactions improve social welfare.

Exercise:

Problem:

Why would a free market never operate at a quantity greater than the equilibrium quantity? *Hint:* What would be required for a transaction to occur at that quantity?

Glossary

consumer surplus
: the extra benefit consumers receive from buying a good or service, measured by what the individuals would have been willing to pay

minus the amount that they actually paid

deadweight loss
the loss in social surplus that occurs when a market produces an inefficient quantity

economic surplus
see social surplus

producer surplus
the extra benefit producers receive from selling a good or service, measured by the price the producer actually received minus the price the producer would have been willing to accept

social surplus
the sum of consumer surplus and producer surplus

Sex and Gender

- Define and differentiate between sex and gender
- Define and discuss what is meant by gender identity
- Understand and discuss the role of homophobia and heterosexism in society
- Distinguish the meanings of transgender, transsexual, and homosexual identities

While the biological differences between males and females are fairly straightforward, the social and cultural aspects of being a man or woman can be complicated. (Photo courtesy of FaceMePLS/flickr)

When filling out a document such as a job application or school registration form you are often asked to provide your name, address, phone number, birth date, and sex or gender. But have you ever been asked to provide your sex *and* your gender? Like most people, you may not have realized that sex and gender are not the same. However, sociologists and most other social scientists view them as conceptually distinct. **Sex** refers to physical or physiological differences between males and females, including both primary sex characteristics (the reproductive system) and secondary

characteristics such as height and muscularity. **Gender** refers to behaviors, personal traits, and social positions that society attributes to being female or male.

A person’s sex, as determined by his or her biology, does not always correspond with his or her gender. Therefore, the terms *sex* and *gender* are not interchangeable. A baby boy who is born with male genitalia will be identified as male. As he grows, however, he may identify with the feminine aspects of his culture. Since the term *sex* refers to biological or physical distinctions, characteristics of sex will not vary significantly between different human societies. Generally, persons of the female sex, regardless of culture, will eventually menstruate and develop breasts that can lactate. Characteristics of gender, on the other hand, may vary greatly between different societies. For example, in U.S. culture, it is considered feminine (or a trait of the female gender) to wear a dress or skirt. However, in many Middle Eastern, Asian, and African cultures, dresses or skirts (often referred to as sarongs, robes, or gowns) are considered masculine. The kilt worn by a Scottish male does not make him appear feminine in his culture.

The dichotomous view of gender (the notion that someone is either male or female) is specific to certain cultures and is not universal. In some cultures gender is viewed as fluid. In the past, some anthropologists used the term *berdache* to refer to individuals who occasionally or permanently dressed and lived as a different gender. The practice has been noted among certain Native American tribes (Jacobs, Thomas, and Lang 1997). Samoan culture accepts what Samoans refer to as a “third gender.” *Fa’afafine*, which translates as “the way of the woman,” is a term used to describe individuals who are born biologically male but embody both masculine and feminine traits. Fa’afafines are considered an important part of Samoan culture. Individuals from other cultures may mislabel them as homosexuals because fa’afafines have a varied sexual life that may include men and women (Poasa 1992).

Note:
The Legalese of Sex and Gender

The terms *sex* and *gender* have not always been differentiated in the English language. It was not until the 1950s that U.S. and British psychologists and other professionals working with intersex and transsexual patients formally began distinguishing between sex and gender. Since then, psychological and physiological professionals have increasingly used the term gender (Moi 2005). By the end of the twenty-first century, expanding the proper usage of the term *gender* to everyday language became more challenging—particularly where legal language is concerned. In an effort to clarify usage of the terms *sex* and *gender*, U.S. Supreme Court Justice Antonin Scalia wrote in a 1994 briefing, “The word gender has acquired the new and useful connotation of cultural or attitudinal characteristics (as opposed to physical characteristics) distinctive to the sexes. That is to say, gender is to sex as feminine is to female and masculine is to male” (*J.E.B. v. Alabama*, 144 S. Ct. 1436 [1994]). Supreme Court Justice Ruth Bader Ginsburg had a different take, however. Viewing the words as synonymous, she freely swapped them in her briefings so as to avoid having the word “sex” pop up too often. It is thought that her secretary supported this practice by suggestions to Ginsberg that “those nine men” (the other Supreme Court justices), “hear that word and their first association is not the way you want them to be thinking” (Case 1995). This anecdote reveals that both sex and gender are actually socially defined variables whose definitions change over time.

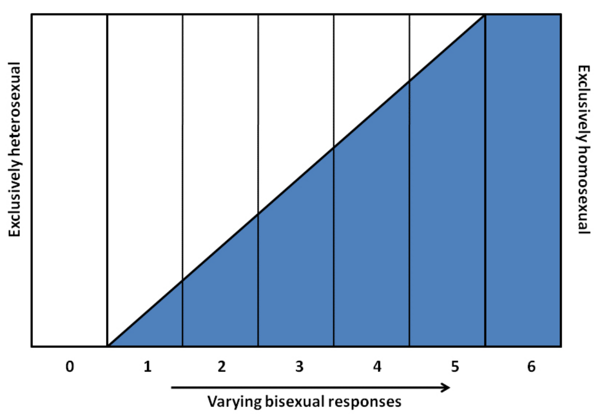
Sexual Orientation

A person’s **sexual orientation** is his or her physical, mental, emotional, and sexual attraction to a particular sex (male or female). Sexual orientation is typically divided into four categories: *heterosexuality*, the attraction to individuals of the other sex; *homosexuality*, the attraction to individuals of the same sex; *bisexuality*, the attraction to individuals of either sex; and *asexuality*, no attraction to either sex. Heterosexuals and homosexuals may also be referred to informally as “straight” and “gay,” respectively. The United States is a **heteronormative society**, meaning it assumes sexual orientation is biologically determined and unambiguous. Consider that homosexuals are often asked, “When did you know you were gay?” but

heterosexuals are rarely asked, “When did you know that you were straight?” (Ryle 2011).

According to current scientific understanding, individuals are usually aware of their sexual orientation between middle childhood and early adolescence (American Psychological Association 2008). They do not have to participate in sexual activity to be aware of these emotional, romantic, and physical attractions; people can be celibate and still recognize their sexual orientation. Homosexual women (also referred to as lesbians), homosexual men (also referred to as gays), and bisexuals of both genders may have very different experiences of discovering and accepting their sexual orientation. At the point of puberty, some may be able to announce their sexual orientations, while others may be unready or unwilling to make their homosexuality or bisexuality known since it goes against U.S. society’s historical norms (APA 2008).

Alfred Kinsey was among the first to conceptualize sexuality as a continuum rather than a strict dichotomy of gay or straight. He created a six-point rating scale that ranges from exclusively heterosexual to exclusively homosexual. See the figure below. In his 1948 work *Sexual Behavior in the Human Male*, Kinsey writes, “Males do not represent two discrete populations, heterosexual and homosexual. The world is not to be divided into sheep and goats … The living world is a continuum in each and every one of its aspects” (Kinsey 1948).



The Kinsey scale indicates that

sexuality can be measured by
more than just heterosexuality
and homosexuality.

Later scholarship by Eve Kosofsky Sedgwick expanded on Kinsey's notions. She coined the term "homosocial" to oppose "homosexual," describing nonsexual same-sex relations. Sedgwick recognized that in U.S. culture, males are subject to a clear divide between the two sides of this continuum, whereas females enjoy more fluidity. This can be illustrated by the way women in the United States can express homosocial feelings (nonsexual regard for people of the same sex) through hugging, handholding, and physical closeness. In contrast, U.S. males refrain from these expressions since they violate the heteronormative expectation that male sexual attraction should be exclusively for females. Research suggests that it is easier for women violate these norms than men, because men are subject to more social disapproval for being physically close to other men (Sedgwick 1985).

There is no scientific consensus regarding the exact reasons why an individual holds a heterosexual, homosexual, or bisexual orientation. Research has been conducted to study the possible genetic, hormonal, developmental, social, and cultural influences on sexual orientation, but there has been no evidence that links sexual orientation to one factor (APA 2008). Research, however, does present evidence showing that homosexuals and bisexuals are treated differently than heterosexuals in schools, the workplace, and the military. In 2011, for example, Sears and Mallory used General Social Survey data from 2008 to show that 27 percent of lesbian, gay, bisexual (LGB) respondents reported experiencing sexual orientation-based discrimination during the five years prior to the survey. Further, 38 percent of openly LGB people experienced discrimination during the same time.

Much of this discrimination is based on stereotypes and misinformation. Some is based on **heterosexism**, which Herek (1990) suggests is both an ideology and a set of institutional practices that privilege heterosexuals and

heterosexuality over other sexual orientations. Much like racism and sexism, heterosexism is a systematic disadvantage embedded in our social institutions, offering power to those who conform to hetereosexual orientation while simultaneously disadvantaging those who do not. *Homophobia*, an extreme or irrational aversion to homosexuals, accounts for further stereotyping and discrimination. Major policies to prevent discrimination based on sexual orientation have not come into effect until the last few years. In 2011, President Obama overturned "don't ask, don't tell," a controversial policy that required homosexuals in the US military to keep their sexuality undisclosed. The Employee Non-Discrimination Act, which ensures workplace equality regardless of sexual orientation, is still pending full government approval. Organizations such as GLAAD (Gay & Lesbian Alliance Against Defamation) advocate for homosexual rights and encourage governments and citizens to recognize the presence of sexual discrimination and work to prevent it. Other advocacy agencies frequently use the acronyms LBGT and LBGTQ, which stands for "Lesbian, Gay, Bisexual, Transgender" (and "Queer" or "Questioning" when the Q is added).

Sociologically, it is clear that gay and lesbian couples are negatively affected in states where they are denied the legal right to marriage. In 1996, The Defense of Marriage Act (**DOMA**) was passed, explicitly limiting the definition of "marriage" to a union between one man and one woman. It also allowed individual states to choose whether or not they recognized same-sex marriages performed in other states. Imagine that you married an opposite-sex partner under similar conditions—if you went on a cross-country vacation the validity of your marriage would change every time you crossed state lines. In another blow to same-sex marriage advocates, in November 2008 California passed Proposition 8, a state law that limited marriage to unions of opposite-sex partners.

Over time, advocates for same-sex marriage have won several court cases, laying the groundwork for legalized same-sex marriage across the United States, including the June 2013 decision to overturn part of DOMA in *Windsor v. United States*, and the Supreme Court's dismissal of *Hollingsworth v. Perry*, affirming the August 2010 ruling that found California's Proposition 8 unconstitutional. In October 2014, the U.S.

Supreme Court declined to hear appeals to rulings against same-sex marriage bans, which effectively legalized same-sex marriage in Indiana, Oklahoma, Utah, Virginia, and Wisconsin, Colorado, North Carolina, West Virginia, and Wyoming (Freedom to Marry, Inc. 2014). Same-sex marriage is now legal across most of the United States. The next few years will determine whether the right to same-sex marriage is affirmed, depending on whether the U.S. Supreme Court takes a judicial step to guarantee the freedom to marry as a civil right.

Gender Roles

As we grow, we learn how to behave from those around us. In this socialization process, children are introduced to certain roles that are typically linked to their biological sex. The term **gender role** refers to society's concept of how men and women are expected to look and how they should behave. These roles are based on norms, or standards, created by society. In U.S. culture, masculine roles are usually associated with strength, aggression, and dominance, while feminine roles are usually associated with passivity, nurturing, and subordination. Role learning starts with socialization at birth. Even today, our society is quick to outfit male infants in blue and girls in pink, even applying these color-coded gender labels while a baby is in the womb.

One way children learn gender roles is through play. Parents typically supply boys with trucks, toy guns, and superhero paraphernalia, which are active toys that promote motor skills, aggression, and solitary play. Daughters are often given dolls and dress-up apparel that foster nurturing, social proximity, and role play. Studies have shown that children will most likely choose to play with "gender appropriate" toys (or same-gender toys) even when cross-gender toys are available because parents give children positive feedback (in the form of praise, involvement, and physical closeness) for gender normative behavior (Caldera, Huston, and O'Brien 1998).

Fathers tend to be more involved when their sons engage in gender-appropriate activities such as sports. (Photo courtesy of Shawn Lea/flickr)

The drive to adhere to masculine and feminine gender roles continues later in life. Men tend to outnumber women in professions such as law enforcement, the military, and politics. Women tend to outnumber men in care-related occupations such as childcare, healthcare (even though the term “doctor” still conjures the image of a man), and social work. These occupational roles are examples of typical U.S. male and female behavior, derived from our culture’s traditions. Adherence to them demonstrates fulfillment of social expectations but not necessarily personal preference (Diamond 2002).

Gender Identity

U.S. society allows for some level of flexibility when it comes to acting out gender roles. To a certain extent, men can assume some feminine roles and

women can assume some masculine roles without interfering with their gender identity. **Gender identity** is a person's deeply held internal perception of his or her gender.

Individuals who identify with the role that is the different from their biological sex are called **transgender**. Transgender is not the same as homosexual, and many homosexual males view both their sex and gender as male. Transgender females are males who have such a strong emotional and psychological connection to the feminine aspects of society that they identify their gender as female. The parallel connection to masculinity exists for transgender males. It is difficult to determine the prevalence of transgenderism in society. However, it is estimated that two to five percent of the U.S. population is transgender (Transgender Law and Policy Institute 2007).

Transgender individuals who attempt to alter their bodies through medical interventions such as surgery and hormonal therapy—so that their physical being is better aligned with gender identity—are called **transsexuals**. They may also be known as male-to-female (MTF) or female-to-male (FTM). Not all transgender individuals choose to alter their bodies: many will maintain their original anatomy but may present themselves to society as another gender. This is typically done by adopting the dress, hairstyle, mannerisms, or other characteristic typically assigned to another gender. It is important to note that people who cross-dress, or wear clothing that is traditionally assigned to a gender different from their biological sex, are not necessarily transgender. Cross-dressing is typically a form of self-expression, entertainment, or personal style, and it is not necessarily an expression against one's assigned gender (APA 2008).

There is no single, conclusive explanation for why people are transgender. Transgender expressions and experiences are so diverse that it is difficult to identify their origin. Some hypotheses suggest biological factors such as genetics or prenatal hormone levels as well as social and cultural factors such as childhood and adulthood experiences. Most experts believe that all of these factors contribute to a person's gender identity (APA 2008).

After years of controversy over the treatment of sex and gender in the *American Psychiatric Association Diagnostic and Statistical Manual for*

Mental Disorders (Drescher 2010), the most recent edition, DSM-5, responds to allegations that the term “Gender Identity Disorder” is stigmatizing by replacing it with “Gender Dysphoria.” Gender Identity Disorder as a diagnostic category stigmatized the patient by implying there was something “disordered” about them. Gender Dysphoria, on the other hand, removes some of that stigma by taking the word "disorder" out while maintaining a category that will protect patient access to care, including hormone therapy and gender reassignment surgery. In the DSM-5, **Gender Dysphoria** is a condition of people whose gender at birth is contrary to the one they identify with. For a person to be diagnosed with Gender Dysphoria, there must be a marked difference between the individual’s expressed/experienced gender and the gender others would assign him or her, and it must continue for at least six months. In children, the desire to be of the other gender must be present and verbalized. This diagnosis is now a separate category from sexual dysfunction and paraphilia, another important part of removing stigma from the diagnosis (APA 2013).

Changing the clinical description may contribute to a larger acceptance of transgender people in society. Studies show that people who identify as transgender are twice as likely to experience assault or discrimination as nontransgender individuals; they are also one and a half times more likely to experience intimidation (National Coalition of Anti-Violence Programs 2010; Giovanniello 2013). Organizations such as the National Coalition of Anti-Violence Programs and Global Action for Trans Equality work to prevent, respond to, and end all types of violence against transgender, transsexual, and homosexual individuals. These organizations hope that by educating the public about gender identity and empowering transgender and transsexual individuals, this violence will end.

Note:

Real-Life *Freaky Friday*

What if you had to live as a sex you were not biologically born to? If you are a man, imagine that you were forced to wear frilly dresses, dainty shoes, and makeup to special occasions, and you were expected to enjoy romantic comedies and daytime talk shows. If you are a woman, imagine

that you were forced to wear shapeless clothing, put only minimal effort into your personal appearance, not show emotion, and watch countless hours of sporting events and sports-related commentary. It would be pretty uncomfortable, right? Well, maybe not. Many people enjoy participating in activities, whether they are associated with their biological sex or not, and would not mind if some of the cultural expectations for men and women were loosened.

Now, imagine that when you look at your body in the mirror, you feel disconnected. You feel your genitals are shameful and dirty, and you feel as though you are trapped in someone else's body with no chance of escape. As you get older, you hate the way your body is changing, and, therefore, you hate yourself. These elements of disconnect and shame are important to understand when discussing transgender individuals. Fortunately, sociological studies pave the way for a deeper and more empirically grounded understanding of the transgender experience.

Chaz Bono is the transgender son of Cher and Sonny Bono. While he was born female, he considers

himself male. Being transgender is not about clothing or hairstyles; it is about self-perception. (Photo courtesy of Greg Hernandez/flickr)

Summary

The terms “sex” and “gender” refer to two different identifiers. Sex denotes biological characteristics differentiating males and females, while gender denotes social and cultural characteristics of masculine and feminine behavior. Sex and gender are not always synchronous. Individuals who strongly identify with the opposing gender are considered transgender.

Section Quiz

Exercise:

Problem:

The terms “masculine” and “feminine” refer to a person’s __________.

a. sex
b. gender
c. both sex and gender
d. none of the above

Solution:
Answer

B

Exercise:

Problem:

The term ________ refers to society's concept of how men and women are expected to act and how they should behave.

a. gender role
b. gender bias
c. sexual orientation
d. sexual attitudes

Solution:
Answer

A

Exercise:

Problem:

Research indicates that individuals are aware of their sexual orientation ________.

a. at infancy
b. in early adolescence
c. in early adulthood
d. in late adulthood

Solution:
Answer

B

Exercise:

Problem:

A person who is biologically female but identifies with the male gender and has undergone surgery to alter her body is considered _______.

a. transgender
b. transsexual
c. a cross-dresser
d. homosexual

Solution:
Answer

B

Exercise:

Problem:

Which of following is correct regarding the explanation for transgenderism?

a. It is strictly biological and associated with chemical imbalances in the brain.
b. It is a behavior that is learned through socializing with other transgender individuals.
c. It is genetic and usually skips one generation.
d. Currently, there is no definitive explanation for transgenderism.

Solution:
Answer

D

Short Answer

Exercise:

Problem:

Why do sociologists find it important to differentiate between sex and gender? What importance does the differentiation have in modern society?

Exercise:

Problem:

How is children's play influenced by gender roles? Think back to your childhood. How "gendered" were the toys and activities available to you? Do you remember gender expectations being conveyed through the approval or disapproval of your playtime choices?

Further Research

For more information on gender identity and advocacy for transgender individuals see the Global Action for Trans Equality web site at http://openstaxcollege.org/l/trans_equality.

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Glossary

DOMA
: Defense of Marriage Act, a 1996 U.S. law explicitly limiting the definition of “marriage” to a union between one man and one woman and allowing each individual state to recognize or deny same-sex marriages performed in other states

gender dysphoria
: a condition listed in the DSM-5 in which people whose gender at birth is contrary to the one they identify with. This condition replaces "gender identity disorder"

gender identity
: a person’s deeply held internal perception of his or her gender

gender role
: society’s concept of how men and women should behave

gender
: a term that refers to social or cultural distinctions of behaviors that are considered male or female

heterosexism
: an ideology and a set of institutional practices that privilege heterosexuals and heterosexuality over other sexual orientations

homophobia
: an extreme or irrational aversion to homosexuals

sex
: a term that denotes the presence of physical or physiological differences between males and females

sexual orientation
: a person's physical, mental, emotional, and sexual attraction to a particular sex (male or female)

transgender
: an adjective that describes individuals who identify with the behaviors and characteristics that are other than their biological sex

transsexuals
: transgender individuals who attempt to alter their bodies through medical interventions such as surgery and hormonal therapy

Dividing Polynomials
In this section, you will:

- Use long division to divide polynomials.
- Use synthetic division to divide polynomials.

Lincoln Memorial, Washington, D.C. (credit: Ron Cogswell, Flickr)

The exterior of the Lincoln Memorial in Washington, D.C., is a large rectangular solid with length 61.5 meters (m), width 40 m, and height 30 m. [footnote] We can easily find the volume using elementary geometry.
National Park Service. "Lincoln Memorial Building Statistics." http://www.nps.gov/linc/historyculture/lincoln-memorial-building-statistics.htm. Accessed 4/3/2014
Equation:

$$\begin{aligned} V &= l \cdot w \cdot h \\ &= 61.5 \cdot 40 \cdot 30 \\ &= 73{,}800 \end{aligned}$$

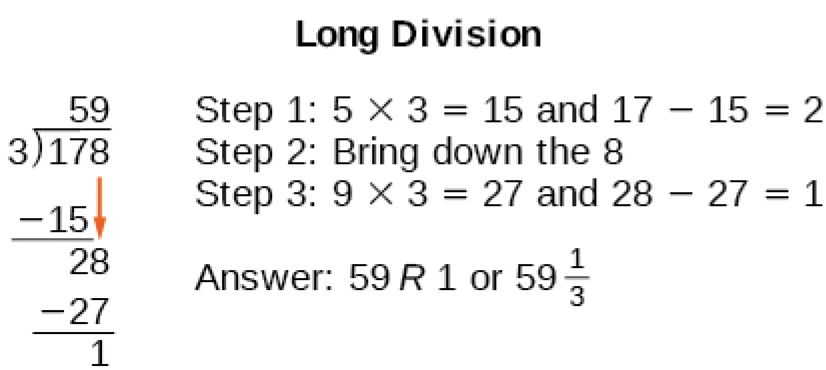
So the volume is 73,800 cubic meters (m^3). Suppose we knew the volume, length, and width. We could divide to find the height.
Equation:

$$\begin{aligned} h &= \frac{V}{l \cdot w} \\ &= \frac{73{,}800}{61.5 \cdot 40} \\ &= 30 \end{aligned}$$

As we can confirm from the dimensions above, the height is 30 m. We can use similar methods to find any of the missing dimensions. We can also use the same method if any or all of the measurements contain variable expressions. For example, suppose the volume of a rectangular solid is given by the polynomial $3x^4 - 3x^3 - 33x^2 + 54x$. The length of the solid is given by $3x$; the width is given by $x - 2$. To find the height of the solid, we can use polynomial division, which is the focus of this section.

Using Long Division to Divide Polynomials

We are familiar with the long division algorithm for ordinary arithmetic. We begin by dividing into the digits of the dividend that have the greatest place value. We divide, multiply, subtract, include the digit in the next place value position, and repeat. For example, let’s divide 178 by 3 using long division.



Another way to look at the solution is as a sum of parts. This should look familiar, since it is the same method used to check division in elementary arithmetic.

Equation:

$$\begin{aligned} \text{dividend} &= (\text{divisor} \;\cdot\; \text{quotient}) + \text{remainder} \\ 178 &= (3 \cdot 59) + 1 \\ &= 177 + 1 \\ &= 178 \end{aligned}$$

We call this the **Division Algorithm** and will discuss it more formally after looking at an example.

Division of polynomials that contain more than one term has similarities to long division of whole numbers. We can write a polynomial dividend as the product of the divisor and the quotient added to the remainder. The terms of the polynomial division correspond to the digits (and place values) of the whole number division. This method allows us to divide two polynomials. For example, if we were to divide $2x^3 - 3x^2 + 4x + 5$ by $x + 2$ using the long division algorithm, it would look like this:

$x+2\overline{)\,2x^3-3x^2+4x+5}$	Set up the division problem.
$\begin{array}{r} 2x^2 \\ x+2\overline{)\,2x^3-3x^2+4x+5} \end{array}$	$2x^3$ divided by x is $2x^2$.
$\begin{array}{r} 2x^2 \\ x+2\overline{)\,2x^3-3x^2+4x+5} \\ \underline{-(2x^3+4x^2)} \\ -7x^2+4x \end{array}$	Multiply $x+2$ by $2x^2$. Subtract.
$\begin{array}{r} 2x^2-7x \\ x+2\overline{)\,2x^3-3x^2+4x+5} \\ \underline{-(2x^3+4x^2)} \\ -7x^2+4x \\ \underline{-(-7x^2+14x)} \\ 18x+5 \end{array}$	Bring down the next term. $-7x^2$ divided by x is $-7x$. Multiply $x+2$ by $-7x$. Subtract. Bring down the next term.
$\begin{array}{r} 2x^2-7x+18 \\ x+2\overline{)\,2x^3-3x^2+4x+5} \\ \underline{-(2x^3+4x^2)} \\ -7x^2+4x \\ \underline{-(-7x^2+14x)} \\ 18x+5 \\ \underline{-18x+36} \\ -31 \end{array}$	$18x$ divided by x is 18. Multiply $x+2$ by 18. Subtract.

We have found

Equation:

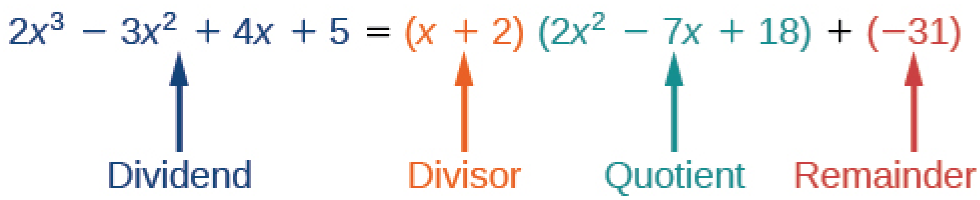
$$\frac{2x^3-3x^2+4x+5}{x+2}=2x^2-7x+18-\frac{31}{x+2}$$

or

Equation:

$$\frac{2x^3-3x^2+4x+5}{x+2}=(x+2)(2x^2-7x+18)-31$$

We can identify the dividend, the divisor, the quotient, and the remainder.



Writing the result in this manner illustrates the Division Algorithm.

Note:

The Division Algorithm

The **Division Algorithm** states that, given a polynomial dividend $f(x)$ and a non-zero polynomial divisor $d(x)$ where the degree of $d(x)$ is less than or equal to the degree of $f(x)$, there exist unique polynomials $q(x)$ and $r(x)$ such that

Equation:

$$f(x)=d(x)q(x)+r(x)$$

$q(x)$ is the quotient and $r(x)$ is the remainder. The remainder is either equal to zero or has degree strictly less than $d(x)$.

If $r(x)=0$, then $d(x)$ divides evenly into $f(x)$. This means that, in this case, both $d(x)$ and $q(x)$ are factors of $f(x)$.

Note: Given a polynomial and a binomial, use long division to divide the polynomial by the binomial.

1. Set up the division problem.
2. Determine the first term of the quotient by dividing the leading term of the dividend by the leading term of the divisor.
3. Multiply the answer by the divisor and write it below the like terms of the dividend.
4. Subtract the bottom binomial from the top binomial.
5. Bring down the next term of the dividend.
6. Repeat steps 2–5 until reaching the last term of the dividend.
7. If the remainder is non-zero, express as a fraction using the divisor as the denominator.

Example:
Exercise:

Problem:
Using Long Division to Divide a Second-Degree Polynomial

Divide $5x^2 + 3x - 2$ by $x + 1$.

Solution:

$x+1\,\overline{)\,5x^2+3x-2}$	Set up division problem.
$\begin{array}{r} 5x \\ x+1\,\overline{)\,5x^2+3x-2} \end{array}$	$5x^2$ divided by x is $5x$.
$\begin{array}{r} 5x \\ x+1\,\overline{)\,5x^2+3x-2} \\ \underline{-(5x^2+5x)} \end{array}$	Multiply $x+1$ by $5x$.
$\begin{array}{r} -2x-2 \end{array}$	Subtract.
$\begin{array}{r} 5x-2 \\ x+1\,\overline{)\,5x^2+3x-2} \\ \underline{-(5x^2+5x)} \\ -2x-2 \end{array}$	Bring down the next term. $-2x$ divided by x is -2.
$\begin{array}{r} \underline{-(-2x-2)} \\ 0 \end{array}$	Multiply $x+1$ by -2. Subtract.

The quotient is $5x - 2$. The remainder is 0. We write the result as

Equation:

$$\frac{5x^2+3x-2}{x+1}=5x-2$$

or

Equation:

$$5x^2+3x-2=(x+1)(5x-2)$$

Analysis

This division problem had a remainder of 0. This tells us that the dividend is divided evenly by the divisor, and that the divisor is a factor of the dividend.

Example:
Exercise:

Problem:
Using Long Division to Divide a Third-Degree Polynomial

Divide $6x^3+11x^2-31x+15$ by $3x-2$.

Solution:

$2x^2 + 5x - 7$	
$3x-2 \overline{)\,6x^3+11x^2-31x+15}$	$6x^3$ divided by $3x$ is $2x^2$.
$-(6x^3-4x^2)$	Multiply $3x-2$ by $2x^2$.
$15x^2-31x$	Subtract. Bring down the next term. $15x^2$ divided by $3x$ is $5x$.
$-(15x^2-10x)$	Multiply $3x-2$ by $5x$.
$-21x+15$	Subtract. Bring down the next term. $-21x$ divided by $3x$ is -7.
$-(-21x+14)$	Multiply $3x-2$ by -7.
1	Subtract. The remainder is 1.

There is a remainder of 1. We can express the result as:

Equation:

$$\frac{6x^3 + 11x^2 - 31x + 15}{3x - 2} = 2x^2 + 5x - 7 + \frac{1}{3x - 2}$$

Analysis

We can check our work by using the Division Algorithm to rewrite the solution. Then multiply.
Equation:

$$(3x - 2)(2x^2 + 5x - 7) + 1 = 6x^3 + 11x^2 - 31x + 15$$

Notice, as we write our result,

- the dividend is $6x^3 + 11x^2 - 31x + 15$
- the divisor is $3x - 2$
- the quotient is $2x^2 + 5x - 7$
- the remainder is 1

Note:
Exercise:

Problem: Divide $16x^3 - 12x^2 + 20x - 3$ by $4x + 5$.

Solution:

$$4x^2 - 8x + 15 - \frac{78}{4x + 5}$$

Using Synthetic Division to Divide Polynomials

As we've seen, long division of polynomials can involve many steps and be quite cumbersome. **Synthetic division** is a shorthand method of dividing polynomials for the special case of dividing by a linear factor whose leading coefficient is 1.

To illustrate the process, recall the example at the beginning of the section.

Divide $2x^3 - 3x^2 + 4x + 5$ by $x + 2$ using the long division algorithm.

The final form of the process looked like this:

$$
\begin{array}{r l}
 & 2x^2 + x + 18 \\
x+2 & \overline{)2x^3 - 3x^2 + 4x + 5} \\
 & \underline{-(2x^3 + 4x^2)} \\
 & -7x^2 + 4x \\
 & \underline{-(-7x^2 - 14x)} \\
 & 18x + 5 \\
 & \underline{-(18x + 36)} \\
 & -31
\end{array}
$$

There is a lot of repetition in the table. If we don't write the variables but, instead, line up their coefficients in columns under the division sign and also eliminate the partial products, we already have a simpler version of the entire problem.

$$
\begin{array}{r|rrrr}
2 & 2 & -3 & 4 & 5 \\
 & \underline{-2} & \underline{-4} & & \\
 & & -7 & 14 & \\
 & & \underline{} & \underline{18} & \underline{-36} \\
 & & & & -31
\end{array}
$$

Synthetic division carries this simplification even a few more steps. Collapse the table by moving each of the rows up to fill any vacant spots. Also, instead of dividing by 2, as we would in division of whole numbers, then multiplying and subtracting the middle product, we change the sign of

the “divisor” to –2, multiply and add. The process starts by bringing down the leading coefficient.

$$\begin{array}{r|rrrr} -2 & 2 & -3 & 4 & 5 \\ & & -4 & 14 & -36 \\ \hline & 2 & -7 & 18 & -31 \end{array}$$

We then multiply it by the “divisor” and add, repeating this process column by column, until there are no entries left. The bottom row represents the coefficients of the quotient; the last entry of the bottom row is the remainder. In this case, the quotient is $2x^2 - 7x + 18$ and the remainder is $-31.$ The process will be made more clear in [link].

Note:
Synthetic Division
Synthetic division is a shortcut that can be used when the divisor is a binomial in the form $x - k.$ In **synthetic division**, only the coefficients are used in the division process.

Note:
Given two polynomials, use synthetic division to divide.

1. Write k for the divisor.
2. Write the coefficients of the dividend.
3. Bring the lead coefficient down.
4. Multiply the lead coefficient by $k.$ Write the product in the next column.
5. Add the terms of the second column.
6. Multiply the result by $k.$ Write the product in the next column.
7. Repeat steps 5 and 6 for the remaining columns.
8. Use the bottom numbers to write the quotient. The number in the last column is the remainder and has degree 0, the next number from the right has degree 1, the next number from the right has degree 2, and so on.

Example:
Exercise:

Problem:
Using Synthetic Division to Divide a Second-Degree Polynomial

Use synthetic division to divide $5x^2 - 3x - 36$ by $x - 3$.

Solution:

Begin by setting up the synthetic division. Write k and the coefficients.

$$\begin{array}{r|rrr} 3 & 5 & -3 & -36 \\ & & & \\ \hline \end{array}$$

Bring down the lead coefficient. Multiply the lead coefficient by k.

$$\begin{array}{r|rrr} 3 & 5 & -3 & -36 \\ & & 15 & \\ \hline & 5 & & \end{array}$$

Continue by adding the numbers in the second column. Multiply the resulting number by k. Write the result in the next column. Then add the numbers in the third column.

$$\begin{array}{r|rrr} 3 & 5 & -3 & -36 \\ & & 15 & 36 \\ \hline & 5 & 12 & 0 \end{array}$$

The result is $5x + 12$. The remainder is 0. So $x - 3$ is a factor of the original polynomial.

Analysis

Just as with long division, we can check our work by multiplying the quotient by the divisor and adding the remainder.

$$(x-3)(5x+12)+0=5x^2-3x-36$$

Example:
Exercise:

Problem:
Using Synthetic Division to Divide a Third-Degree Polynomial

Use synthetic division to divide $4x^3+10x^2-6x-20$ by $x+2$.

Solution:

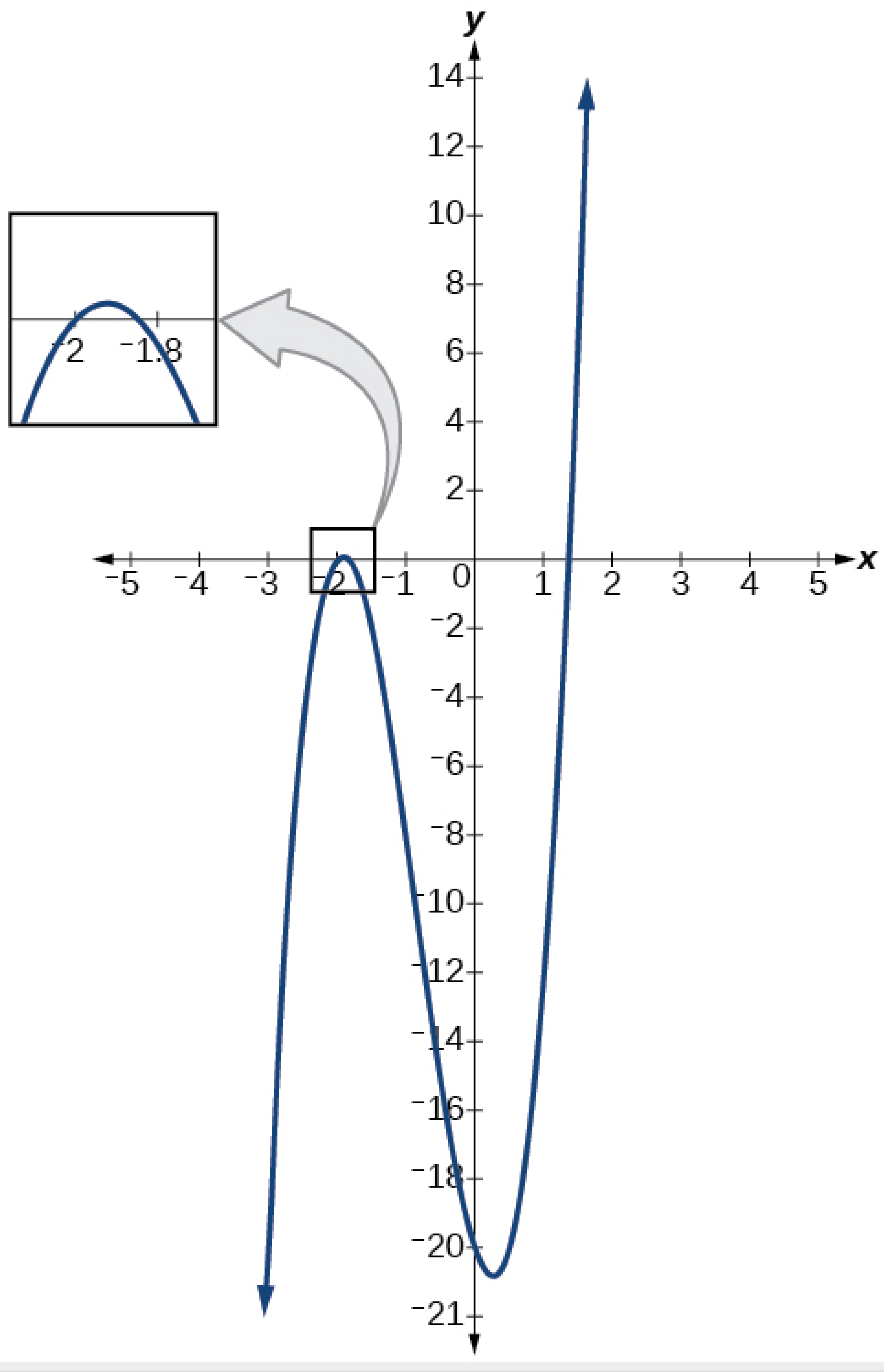
The binomial divisor is $x+2$ so $k=-2$. Add each column, multiply the result by –2, and repeat until the last column is reached.

−2	4	10	−6	−20
		−8	−4	20
	4	2	−10	0

The result is $4x^2+2x-10$. The remainder is 0. Thus, $x+2$ is a factor of $4x^3+10x^2-6x-20$.

Analysis

The graph of the polynomial function $f(x)=4x^3+10x^2-6x-20$ in [link] shows a zero at $x=k=-2$. This confirms that $x+2$ is a factor of $4x^3+10x^2-6x-20$.



Example:
Exercise:

Problem:
Using Synthetic Division to Divide a Fourth-Degree Polynomial

Use synthetic division to divide $-9x^4+10x^3+7x^2-6$ by $x-1$.

Solution:

Notice there is no *x*-term. We will use a zero as the coefficient for that term.

$$\begin{array}{r|rrrrr} 1 & -9 & 10 & 7 & 0 & -6 \\ & & -9 & 1 & 8 & 8 \\ \hline & -9 & 1 & 8 & 8 & 2 \end{array}$$

The result is $-9x^3+x^2+8x+8+\frac{2}{x-1}$.

Note:
Exercise:

Problem:

Use synthetic division to divide $3x^4+18x^3-3x+40$ by $x+7$.

Solution:

$3x^3-3x^2+21x-150+\frac{1{,}090}{x+7}$

Using Polynomial Division to Solve Application Problems

Polynomial division can be used to solve a variety of application problems involving expressions for area and volume. We looked at an application at

the beginning of this section. Now we will solve that problem in the following example.

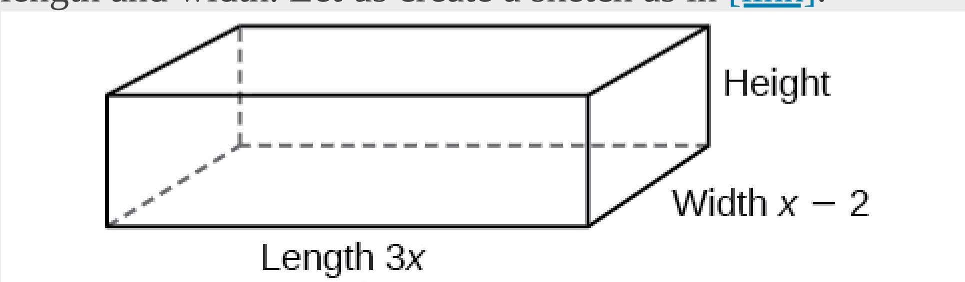
Example:
Exercise:

Problem:
Using Polynomial Division in an Application Problem

The volume of a rectangular solid is given by the polynomial $3x^4 - 3x^3 - 33x^2 + 54x.$ The length of the solid is given by $3x$ and the width is given by $x - 2.$ Find the height of the solid.

Solution:

There are a few ways to approach this problem. We need to divide the expression for the volume of the solid by the expressions for the length and width. Let us create a sketch as in [link].



We can now write an equation by substituting the known values into the formula for the volume of a rectangular solid.
Equation:

$$\begin{aligned} V &= l \cdot w \cdot h \\ 3x^4 - 3x^3 - 33x^2 + 54x &= 3x \cdot (x - 2) \cdot h \end{aligned}$$

To solve for $h,$ first divide both sides by $3x.$
Equation:

$$\frac{3x\cdot(x-2)\cdot h}{3x} = \frac{3x^4-3x^3-33x^2+54x}{3x}$$
$$(x-2)h = x^3 - x^2 - 11x + 18$$

Now solve for h using synthetic division.

Equation:

$$h = \frac{x^3 - x^2 - 11x + 18}{x - 2}$$

Equation:

$$\begin{array}{r|rrrr} & 1 & -1 & -11 & 18 \\ 2 & & 2 & 2 & -18 \\ \hline & 1 & 1 & -9 & 0 \end{array}$$

The quotient is $x^2 + x - 9$ and the remainder is 0. The height of the solid is $x^2 + x - 9$.

Note:
Exercise:

Problem:

The area of a rectangle is given by $3x^3 + 14x^2 - 23x + 6.$ The width of the rectangle is given by $x + 6.$ Find an expression for the length of the rectangle.

Solution:

$3x^2 - 4x + 1$

Note:
Access these online resources for additional instruction and practice with polynomial division.

- Dividing a Trinomial by a Binomial Using Long Division
- Dividing a Polynomial by a Binomial Using Long Division
- Ex 2: Dividing a Polynomial by a Binomial Using Synthetic Division
- Ex 4: Dividing a Polynomial by a Binomial Using Synthetic Division

Key Equations

Division Algorithm	$f(x) = d(x)q(x) + r(x)$ where $q(x) \neq 0$

Key Concepts

- Polynomial long division can be used to divide a polynomial by any polynomial with equal or lower degree. See [link] and [link].
- The Division Algorithm tells us that a polynomial dividend can be written as the product of the divisor and the quotient added to the remainder.
- Synthetic division is a shortcut that can be used to divide a polynomial by a binomial in the form $x - k.$ See [link], [link], and [link].
- Polynomial division can be used to solve application problems, including area and volume. See [link].

Section Exercises

Verbal

Exercise:

Problem:

If division of a polynomial by a binomial results in a remainder of zero, what can be conclude?

Solution:

The binomial is a factor of the polynomial.

Exercise:

Problem:

If a polynomial of degree n is divided by a binomial of degree 1, what is the degree of the quotient?

Algebraic

For the following exercises, use long division to divide. Specify the quotient and the remainder.

Exercise:

Problem: $\left(x^2+5x-1\right) \div (x-1)$

Solution:

$x+6+\frac{5}{x-1}$, quotient: $x+6$, remainder: 5

Exercise:

Problem: $\left(2x^2-9x-5\right) \div (x-5)$

Exercise:

Problem: $\left(3x^2+23x+14\right) \div (x+7)$

Solution:

$3x+2$, quotient: $3x+2$, remainder: 0

Exercise:

Problem: $\left(4x^2-10x+6\right) \div (4x+2)$

Exercise:

Problem: $\left(6x^2-25x-25\right) \div (6x+5)$

Solution:

$x-5$, quotient: $x-5$, remainder: 0

Exercise:

Problem: $\left(-x^2-1\right) \div (x+1)$

Exercise:

Problem: $\left(2x^2-3x+2\right) \div (x+2)$

Solution:

$2x-7+\frac{16}{x+2}$, quotient: $2x-7$, remainder: 16

Exercise:

Problem: $\left(x^3-126\right) \div (x-5)$

Exercise:

Problem: $\left(3x^2-5x+4\right) \div (3x+1)$

Solution:

$x-2+\frac{6}{3x+1}$, quotient: $x-2$, remainder: 6

Exercise:

Problem: $\left(x^3-3x^2+5x-6\right)\div(x-2)$

Exercise:

Problem: $\left(2x^3+3x^2-4x+15\right)\div(x+3)$

Solution:

$2x^2-3x+5$, quotient: $2x^2-3x+5$, remainder: 0

For the following exercises, use synthetic division to find the quotient.

Exercise:

Problem: $\left(3x^3-2x^2+x-4\right)\div(x+3)$

Exercise:

Problem: $\left(2x^3-6x^2-7x+6\right)\div(x-4)$

Solution:

$2x^2+2x+1+\frac{10}{x-4}$

Exercise:

Problem: $\left(6x^3-10x^2-7x-15\right)\div(x+1)$

Exercise:

Problem: $\left(4x^3-12x^2-5x-1\right)\div(2x+1)$

Solution:

$2x^2 - 7x + 1 - \frac{2}{2x+1}$

Exercise:

Problem: $\left(9x^3 - 9x^2 + 18x + 5\right) \div (3x - 1)$

Exercise:

Problem: $\left(3x^3 - 2x^2 + x - 4\right) \div (x + 3)$

Solution:

$3x^2 - 11x + 34 - \frac{106}{x+3}$

Exercise:

Problem: $\left(-6x^3 + x^2 - 4\right) \div (2x - 3)$

Exercise:

Problem: $\left(2x^3 + 7x^2 - 13x - 3\right) \div (2x - 3)$

Solution:

$x^2 + 5x + 1$

Exercise:

Problem: $\left(3x^3 - 5x^2 + 2x + 3\right) \div (x + 2)$

Exercise:

Problem: $\left(4x^3 - 5x^2 + 13\right) \div (x + 4)$

Solution:

$4x^2 - 21x + 84 - \frac{323}{x+4}$

Exercise:

Problem: $\left(x^3 - 3x + 2\right) \div (x + 2)$

Exercise:

Problem: $\left(x^3 - 21x^2 + 147x - 343\right) \div (x - 7)$

Solution:

$x^2 - 14x + 49$

Exercise:

Problem: $\left(x^3 - 15x^2 + 75x - 125\right) \div (x - 5)$

Exercise:

Problem: $\left(9x^3 - x + 2\right) \div (3x - 1)$

Solution:

$3x^2 + x + \frac{2}{3x-1}$

Exercise:

Problem: $\left(6x^3 - x^2 + 5x + 2\right) \div (3x + 1)$

Exercise:

Problem: $\left(x^4 + x^3 - 3x^2 - 2x + 1\right) \div (x + 1)$

Solution:

$x^3 - 3x + 1$

Exercise:

Problem: $\left(x^4 - 3x^2 + 1\right) \div (x - 1)$

Exercise:

Problem: $\left(x^4 + 2x^3 - 3x^2 + 2x + 6\right) \div (x + 3)$

Solution:

$x^3 - x^2 + 2$

Exercise:

Problem: $\left(x^4 - 10x^3 + 37x^2 - 60x + 36\right) \div (x - 2)$

Exercise:

Problem: $\left(x^4 - 8x^3 + 24x^2 - 32x + 16\right) \div (x - 2)$

Solution:

$x^3 - 6x^2 + 12x - 8$

Exercise:

Problem: $\left(x^4 + 5x^3 - 3x^2 - 13x + 10\right) \div (x + 5)$

Exercise:

Problem: $\left(x^4 - 12x^3 + 54x^2 - 108x + 81\right) \div (x - 3)$

Solution:

$x^3 - 9x^2 + 27x - 27$

Exercise:

Problem: $\left(4x^4 - 2x^3 - 4x + 2\right) \div (2x - 1)$

Exercise:

Problem: $\left(4x^4 + 2x^3 - 4x^2 + 2x + 2\right) \div (2x + 1)$

Solution:

$2x^3 - 2x + 2$

For the following exercises, use synthetic division to determine whether the first expression is a factor of the second. If it is, indicate the factorization.

Exercise:

Problem: $x - 2,\ 4x^3 - 3x^2 - 8x + 4$

Exercise:

Problem: $x - 2,\ 3x^4 - 6x^3 - 5x + 10$

Solution:

Yes $(x - 2)(3x^3 - 5)$

Exercise:

Problem: $x + 3,\ -4x^3 + 5x^2 + 8$

Exercise:

Problem: $x - 2,\ 4x^4 - 15x^2 - 4$

Solution:

Yes $(x - 2)(4x^3 + 8x^2 + x + 2)$

Exercise:

Problem:$x-\frac{1}{2}$, $2x^4-x^3+2x-1$

Exercise:

Problem:$x+\frac{1}{3}$, $3x^4+x^3-3x+1$

Solution:

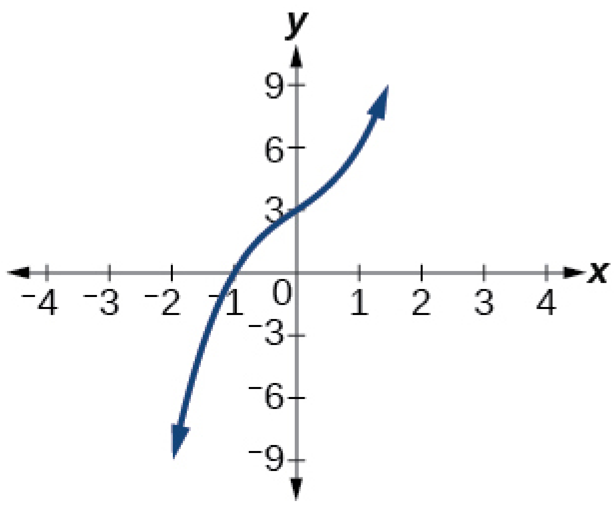
No

Graphical

For the following exercises, use the graph of the third-degree polynomial and one factor to write the factored form of the polynomial suggested by the graph. The leading coefficient is one.

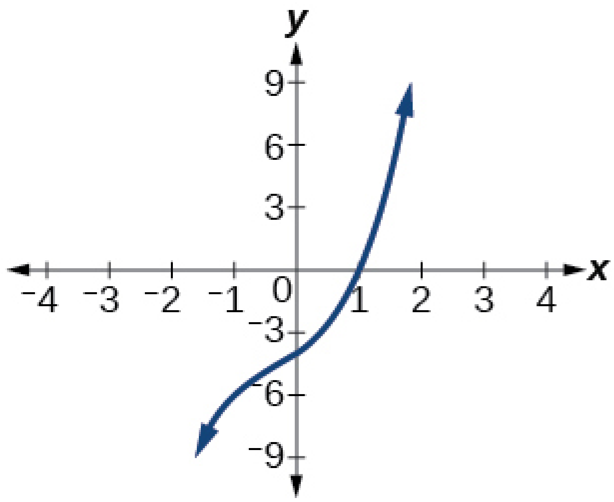
Exercise:

Problem: Factor is x^2-x+3



Exercise:

Problem: Factor is $(x^2 + 2x + 4)$

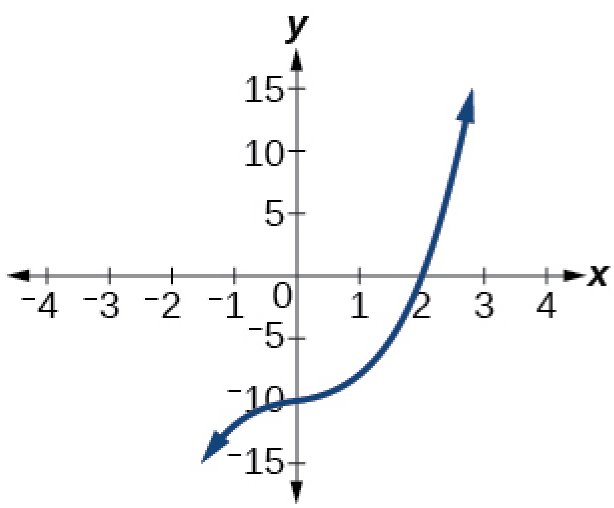


Solution:

$(x - 1)(x^2 + 2x + 4)$

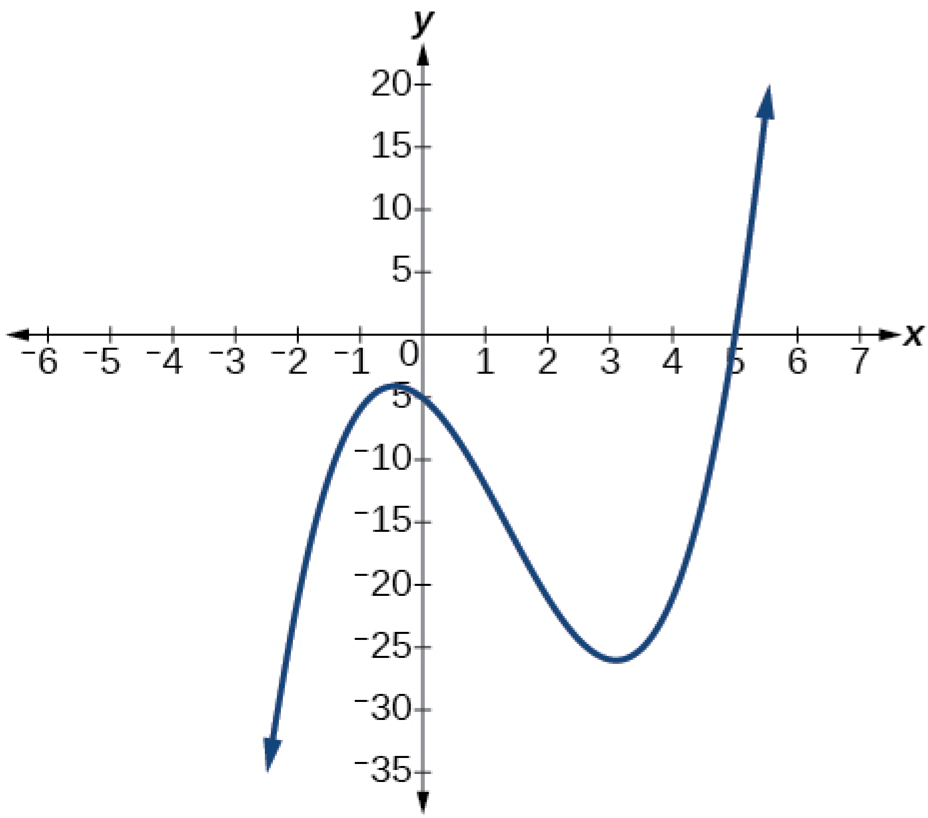
Exercise:

Problem: Factor is $x^2 + 2x + 5$



Exercise:

Problem: Factor is $x^2 + x + 1$

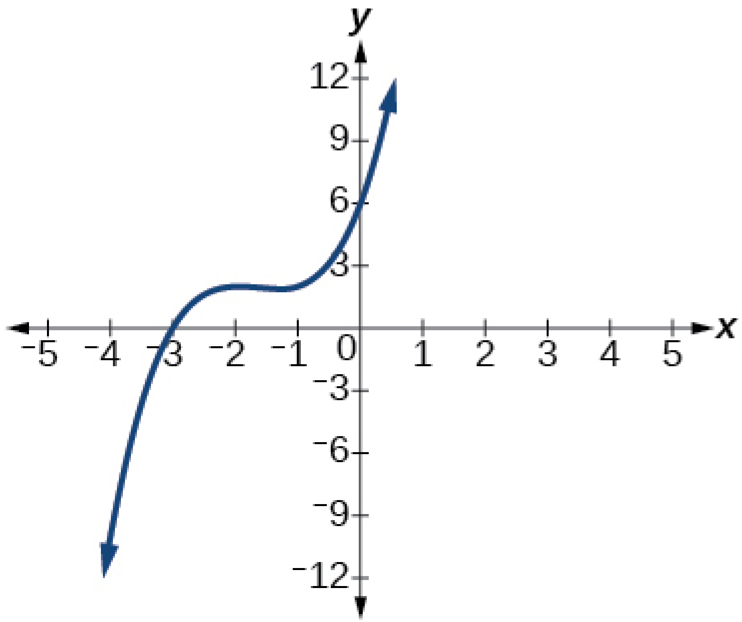


Solution:

$(x - 5)(x^2 + x + 1)$

Exercise:

Problem: Factor is$x^2 + 2x + 2$



For the following exercises, use synthetic division to find the quotient and remainder.

Exercise:

Problem: $\frac{4x^3-33}{x-2}$

Solution:

$\text{Quotient: } 4x^2+8x+16, \text{remainder: } -1$

Exercise:

Problem: $\frac{2x^3+25}{x+3}$

Exercise:

Problem: $\frac{3x^3+2x-5}{x-1}$

Solution:

$\text{Quotient: } 3x^2+3x+5, \text{remainder: } 0$

Exercise:

Problem: $\frac{-4x^3-x^2-12}{x+4}$

Exercise:

Problem: $\frac{x^4-22}{x+2}$

Solution:

$\text{Quotient:}\,x^3-2x^2+4x-8,\text{remainder:}\,-6$

Technology

For the following exercises, use a calculator with CAS to answer the questions.

Exercise:

Problem:

Consider $\frac{x^k-1}{x-1}$ with $k=1,\ 2,\ 3$. What do you expect the result to be if $k=4$?

Exercise:

Problem:

Consider $\frac{x^k+1}{x+1}$ for $k=1,\ 3,\ 5$. What do you expect the result to be if $k=7$?

Solution:

$x^6-x^5+x^4-x^3+x^2-x+1$

Exercise:

Problem:

Consider $\frac{x^4-k^4}{x-k}$ for $k=1,\ 2,\ 3$. What do you expect the result to be if $k=4$?

Exercise:

Problem:

Consider $\frac{x^k}{x+1}$ with $k=1,\ 2,\ 3$. What do you expect the result to be if $k=4$?

Solution:

$x^3-x^2+x-1+\frac{1}{x+1}$

Exercise:

Problem:

Consider $\frac{x^k}{x-1}$ with $k=1,\ 2,\ 3$. What do you expect the result to be if $k=4$?

Extensions

For the following exercises, use synthetic division to determine the quotient involving a complex number.

Exercise:

Problem: $\frac{x+1}{x-i}$

Solution:

$1+\frac{1+i}{x-i}$

Exercise:

Problem: $\frac{x^2+1}{x-i}$

Exercise:

Problem: $\frac{x+1}{x+i}$

Solution:

$1 + \frac{1-i}{x+i}$

Exercise:

Problem: $\frac{x^2+1}{x+i}$

Exercise:

Problem: $\frac{x^3+1}{x-i}$

Solution:

$x^2 - ix - 1 + \frac{1-i}{x-i}$

Real-World Applications

For the following exercises, use the given length and area of a rectangle to express the width algebraically.

Exercise:

Problem: Length is $x+5,$ area is $2x^2+9x-5.$

Exercise:

Problem: Length is $2x\ +\ 5,$ area is $4x^3+10x^2+6x+15$

Solution:

$2x^2 + 3$

Exercise:

Problem: Length is $3x–4,$ area is $6x^4 - 8x^3 + 9x^2 - 9x - 4$

For the following exercises, use the given volume of a box and its length and width to express the height of the box algebraically.

Exercise:

Problem:

Volume is $12x^3 + 20x^2 - 21x - 36,$ length is $2x + 3,$ width is $3x - 4.$

Solution:

$2x + 3$

Exercise:

Problem:

Volume is $18x^3 - 21x^2 - 40x + 48,$ length is $3x–4,$ width is $3x–4.$

Exercise:

Problem:

Volume is $10x^3 + 27x^2 + 2x - 24,$ length is $5x–4,$ width is $2x + 3.$

Solution:

$x + 2$

Exercise:

Problem:

Volume is $10x^3+30x^2-8x-24,$ length is $2,$ width is $x+3.$

For the following exercises, use the given volume and radius of a cylinder to express the height of the cylinder algebraically.

Exercise:

Problem: Volume is $\pi(25x^3-65x^2-29x-3),$ radius is $5x+1.$

Solution:

$x-3$

Exercise:

Problem: Volume is $\pi(4x^3+12x^2-15x-50),$ radius is $2x+5.$

Exercise:

Problem:

Volume is $\pi(3x^4+24x^3+46x^2-16x-32),$ radius is $x+4.$

Solution:

$3x^2-2$

Glossary

Division Algorithm
: given a polynomial dividend $f(x)$ and a non-zero polynomial divisor $d(x)$ where the degree of $d(x)$ is less than or equal to the degree of $f(x),$ there exist unique polynomials $q(x)$ and $r(x)$ such that $f(x)=d(x)q(x)+r(x)$ where $q(x)$ is the quotient and $r(x)$ is the remainder. The remainder is either equal to zero or has degree strictly less than $d(x).$

synthetic division

a shortcut method that can be used to divide a polynomial by a binomial of the form $x - k$